Simplified Approach for Pile and Foundation Interaction Analysis[a]

Discussion by George Gazetas,[3] Member, ASCE

The paper studies the lateral dynamic response of a single pile in a homogeneous half-space and a homogeneous stratum. Both the theoretical development and the results of the paper raise some questions which follow in order of importance.

First, the pile is treated as a *shear*, rather than a flexural, beam. This, clearly, is an unrealistic assumption. What is not clear is the reason that prompted the authors to introduce it. The equations of motion can, after all, be easily solved even for a pile deforming in bending (flexural beam).

Second, the results presented in Fig. 6(b) and Fig. 8, for a pile floating in a homogeneous half-space, show the existence of a resonant frequency of about

$$a_o = a_{o,r} \approx 0.045 \tag{37}$$

At this frequency, the horizontal stiffness (real part of the impedance) drops to about 60% of its static value; at lower frequencies, radiation damping (represented by the imaginary part of the impedance) vanishes.

This is a very surprising and, in fact, puzzling behavior. In a homogeneous half-space, a floating pile is expected to have stiffness and damping that are smooth functions of frequency, with neither resonance nor cutoff frequency, as has been shown by Roesset and Angelides (1979), Roesset (1980), Kaynia (1981), Gazetas and Dobry (1984), Banerjee and Sen (1987), Pak and Jennings (1987). The authors do not offer any discussion at all of their results in Figs. 6(b) and 8. Is, perhaps, this surprising behavior of their system an artifact of the unrealistic shear-beam assumption, of inappropriate modeling of the lateral wave transmission, or of some other, so far unknown, phenomenon?

In any case, the authors should also provide some comparisons of their results with the established dynamic solutions, both for a stratum and a half-space, to demonstrate the accuracy of their method.

Third, in using the "Vlasov" model to represent the soil reactions against the oscillating pile, it appears that the authors might not have been aware of the seminal work on the subject by Kerr (1964, 1965, 1984) or of several applications of the model in soil-structure interaction problems (Jones and Xenophontos 1977; Selvadurai 1979; Gazetas 1982; Trochanis et al. 1987). Of particular interest is the article of Kerr (1984), in which he proved that the Vlasov soil model coincides with the two-parameter "Pasternak" model. The latter models the soil supporting a foundation as a series of Winkler springs, the tops of which (forming the soil-foundation interface) are tied together with a shear beam. The second-order governing differential equations can then be written directly, with no need to resort to variational calculus as is done in the paper. (Incidentally, this soil model should not be called a "linear elastic material" as the authors do in their abstract, since the established meaning of this term is obviously different).

Fourth, the effect of material damping ratio is also unusual. To begin with, the factor δ in (3*b*) should not be called the "hysteretic material damping ratio for the soil." With the accepted definition in the geotechnical literature, δ is *two* times the hysteretic damping ratio in the soil, which is hereafter called ξ. In any case, the results of the paper are for $\delta = 0.20$, i.e., $\xi = 10\%$ soil damping. With such a (high) damping, it is not possible to have exactly zero imaginary part of the pile stiffness below the cutoff frequency. In fact, the overall hysteretic damping ratio D in the pile-soil system at such low frequencies would be approximately equal to 3/4 of the damping ratio in the soil, as shown by Dobry et al. (1982), Velez and Gazetas (1983), and others. (The exact value is not always 3/4 but is a function of the relative rigidity of the pile, approaching 1, i.e. overall damping equal to that in the soil, for a perfectly rigid pile.) Thus, instead of zero, the low-frequency ordinate of all imaginary parts in Fig. 6 should be approximately equal to

$$\text{Im}(K_u)/K_u(0) \equiv 2D \approx 2 \times \frac{3}{4} \times 0.10 = 0.15 \tag{38}$$

APPENDIX. REFERENCES

Banerjee, P. K., and Sen, R. (1987). "Dynamic behavior of axially and laterally loaded piles and pile groups." *Dynamic Behavior of Foundations and Buried Structures*, Elsevier Applied Science, New York, N.Y., 95–133.

[a] September, 1993, Vol. 119, No. 9, by Kiming Sun and Jose A. Pires (Paper 2201).

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Dobry, R., et al. (1982). "Horizontal stiffness and damping of single piles." *J. Geotech. Engrg.*, ASCE, 108(3), 439–459.
Gazetas, G. (1982). "Progressive collapse of rigid plastic circular foundations." *J. Engrg. Mech.*, ASCE, 108(3), 493–508.
Gazetas, G., and Dobry, R. (1984). "Horizontal response of piles in layered soil." *J. Geotech. Engrg.*, ASCE, 110(1), 20–40.
Jones, R., and Xenophontos, J. (1977). "The Vlasov foundation mode." *Int. J. Mech. Sci.*, 19, 317–323.
Kaynia, A. (1981). "Dynamic stiffness and seismic response of pile groups." *Res. Rep.*, Massachusetts Institute of Technology, Cambridge, Mass.
Kerr, A. D. (1964). "Elastic and viscoelastic foundation models." *J. Appl. Mech.*, 32, 491–498.
Kerr, A. D. (1965). "A study of a new foundation model." *Acta Mechanica*, 1, 135–147.
Kerr, A. D. (1984). "On the formal development of elastic foundation models." *Ingenieur Archiv.*, 54, 455–464.
Pak, R. Y. S., and Jennings, P. C. (1987). "Elastodynamic response of pile in transverse excitations." *J. Engrg. Mech.*, ASCE, 113(7), 1101–1116.
Roesset, J. M. (1980). "Stiffness and damping coefficients of foundations." *Dynamic response of pile foundations*, M. O'Neil and R. Dobry, eds., ASCE, New York, N.Y., 1–30.
Roesset, J. M., and Angelides, D. (1979). "Dynamic stiffness and piles." *Numerical methods in offshore piling*, Institution of Civil Engineers, London, U.K., 75–81.
Selvadurai, A. P. S. (1979). *Elastic analysis of soil-foundation interaction*. Elsevier, Amsterdam, The Netherlands.
Trochanis, A. M., Chellilah, R., and Bielak, J. (1987). "Unified approach for beams on elastic foundations under moving loads." *J. Geotech. Engrg.*, ASCE, 113(8), 879–895.
Velez, A., and Gazetas, G. (1983). "Lateral dynamic response of constrained-head piles." *J. Geotech. Engrg.*, ASCE, 109(8), 1063–1081.

Closure by Keming Sun,[4] and Jose A. Pires,[5] Associate Members, ASCE

The discusser raises some questions pertaining to the theoretical developments and the results of the paper. Those questions are addressed as follows.

First, the method is introduced for a pile treated as a shear beam that approximates the response of a short elastic pile with no rotations allowed at the pile head and pile tip, a situation that may occur for a pile clamped to a rigid bottom underlying a soil layer. Otherwise, the writers agree with the discusser that approximating the pile behavior with that of a shear beam may not be a satisfactory approximation. Nevertheless, the method and formulation presented are general and can be extended to flexural piles as written in the introduction and conclusions of the paper, and by the discusser.

Second, the writers agree with the discusser that the real and imaginary parts of the dynamic stiffness of floating piles in a half-space are expected (at least for the swaying mode) to be smooth functions of frequency without a cutoff frequency. The cutoff frequency shown in Fig. 6(b) is, possibly, a result of the shear beam assumption for the pile.

Comparison with existing results, the relationship between the response of the pile head and the loads applied at the pile head can be expressed by [e.g. Pak and Jennings (1987)]

$$\begin{pmatrix} \Delta \\ \theta \end{pmatrix} = \begin{bmatrix} C_{vv} & C_{vm} \\ C_{mv} & C_{mm} \end{bmatrix} \begin{pmatrix} \bar{V}_0 \\ \bar{M}_0 \end{pmatrix} \tag{39}$$

where Δ = horizontal displacement of the pile head; θ = pile-head rotation, $\bar{V}_0$ = normalized applied horizontal force; $\bar{M}_0$ = normalized applied moment; and the elements of the compliance matrix [i.e. the matrix in (39)] are functions of the loading frequency. The displacement Δ for $\theta = 0$ is given by

$$\Delta = \frac{C_{vv}C_{mm} - C_{vm}C_{mv}}{C_{mm}} \bar{V}_0 = C_{VV}\bar{V}_0 \tag{40}$$

The real and imaginary part of the compliance C_{VV} normalized with respect to its static value, and computed using the results given in Kuhlemeyer (1979a), for $E_p/E_s = 567$ and $\delta = 0.0$, and in Pak and Jennings (1987), for $E_p/E_s = 1{,}000$ (the results available for $E_p/E_s = 567$ are not sufficient to compute C_{VV}), $H/R_0 = 50$ and $\delta = 0.0$, are shown in Fig. 10. To compute the results shown in Fig. 10, the static values of the compliances are needed. Those values are computed here using the method proposed by Kuhlemeyer (1979a), i.e., equations 12 and 20 in Kuhlemeyer (1979a), for both sets of curves. Also shown in Fig. 10 are the real and imaginary part of the compliance function, C_{VV}, computed with the method proposed by the writers, for $H/R_0 = 50$, $E_p/E_s = 567$, $\delta = 0.00$, and $\delta = 0.02$, which treats the pile as a shear beam.

The method presented by the writers indicates a cutoff frequency and a peak in the real part

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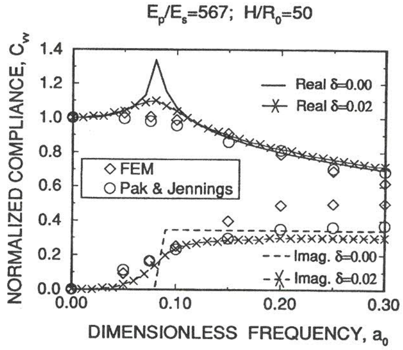

FIG. 10. Comparison with Existing Results for Floating Pile in Half-Space (E_p/E_s = 1,000 for Curves Labeled Pak and Jennings)

of the flexibility coefficient, which are not observed with the other methods. It is noted that the results labeled Pak and Jennings are supposed to be the more accurate ones, provided that the static compliances are adequately computed using the method proposed by Kuhlemeyer (1979a).

Third, the discusser provides additional references dealing with the so-called Vlasov and Leontiev method. While providing an additional background for the Vlasov and Leontiev method, those references are not relevant for the development and presentation of the method described in the paper. In particular, the discusser writes that the Vlasov and Leontiev soil model coincides with the so-called Pasternak soil model (Kerr 1984) for which the second-order differential equations can be written directly, without the need to resort to variational calculus.

The formulation presentation in the paper makes use of variational calculus, which leads to a full description of the pile-soil interaction in terms of the interaction parameter γ, which is obtained iteratively for the static case and with the technique proposed in the paper for the dynamic case. The use of variational calculus is generally considered a highly desirable and preferred approach to establish the governing differential equations for both static and dynamic problems.

Fourth, the material damping ratio constant δ used in the paper is defined, without ambiguity, following (3*b*) in the paper. The results presented in Fig. 6 are for $\delta = 0.02$ as written near the bottom of page 1475, and not $\delta = 0.20$ as erroneously marked in Fig. 6. Therefore, the results shown in Fig. 6 are for a hysteretic material damping ratio $\xi = 0.01$ (using the discusser's notation). For such small hysteretic damping ratios for the soil, and for a short stiff pile ($H/R_0 = 20$) very small values may be expected for the imaginary part of the dynamic soil stiffness.

Errata. The following corrections should be made to the original paper:

Page 1473, Fig. 6(a): should read "$\delta = 0.02$" instead of "$\delta = 0.2$"

Page 1473, Fig. 6(b): should read "$\delta = 0.02$" instead of "$\delta = 0.2$"

Page 1471, Eq. (33): Should read

$$\frac{n}{m} = \cdots = \frac{\rho_s}{G_s} \frac{\int_0^\infty G_s \left[\frac{dF(z)}{dz}\right]^2 dz}{\int_0^\infty \rho_s F^2(z)\, dz} \tag{33}$$

instead of

$$\frac{n}{m} = \cdots = \frac{\rho_s}{G_s} \frac{\int_0^\infty G_s \left[\frac{dF(z)}{dz}\right]^2 dz}{\rho_s F^2(z)\, dz} \tag{33}$$